$$S^2 = \sum_{i=1}^{n} \frac{(x_i - \bar{x})^2}{n-1}$$

$$n \sum_{i=1}^{n} (x_i)^2 - \left(\sum_{i=1}^{n} x_i\right)^2 \stackrel{?}{=} \sum_{i=1}^{n} (x_i - \bar{x})^2$$

$$= \sum_{i=1}^{n} \left(x_i^2 - 2x_i\bar{x} + \bar{x}^2\right)$$

$$= \sum_{i=1}^{n} x_i^2 - \sum_{i=1}^{n} 2x_i\bar{x} + \sum_{i=1}^{n} \bar{x}^2$$

$$\frac{1}{n}\sum_{i=1}^{n} x_i = \bar{x}$$

$$\sum_{i=1}^{n} x_i = n\bar{x}$$

$$= \sum_{i=1}^{n} x_i^2 - 2\bar{x} \sum_{i=1}^{n} x_i + \bar{x}^2 \cdot \sum_{i=1}^{n} 1$$

$$= \sum_{i=1}^{n} x_i^2 - 2\bar{x} \cdot n\bar{x} + \bar{x}^2 \cdot n$$

$$= \sum_{i=1}^{n} x_i^2 - 2n\bar{x}^2 + \bar{x}^2 \cdot n$$

$$= \sum_{i=1}^{n} x_i^2 - n\bar{x}^2$$

$$= \sum_{i=1}^{n} x_i^2 - n\frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n^2}$$

$$= \sum_{i=1}^{n} x_i^2 - \frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n} \quad (\cdot n)$$

$$\stackrel{!}{=} n\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2$$