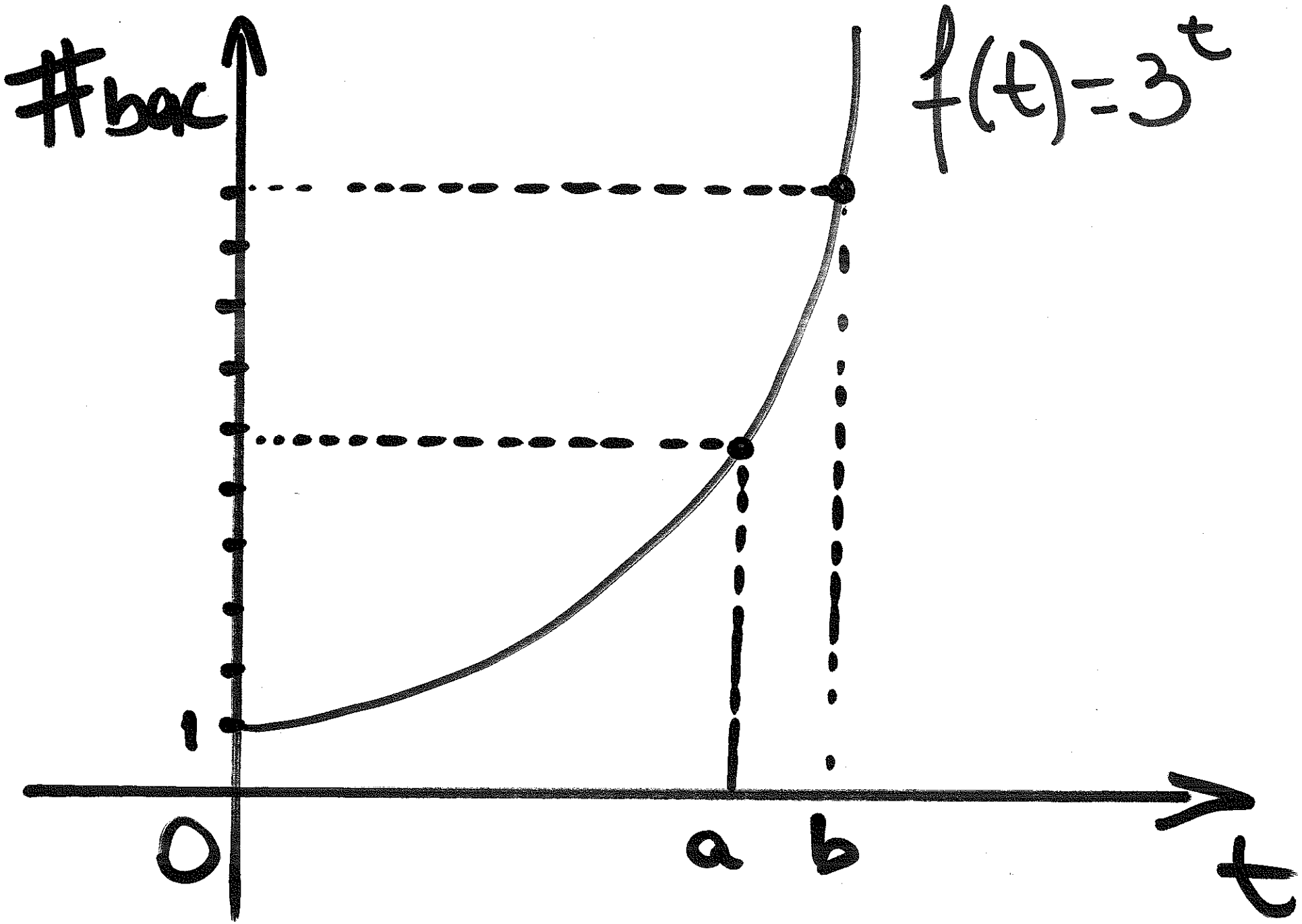
UNIDAD 4: FUNCIONES EXPONENCIALES Y LOGARITMICAS

$$f(x) = 2^x$$

$$f(3) = 2^3 = 8$$

$$f(10) = 2^{10} = 1024$$

$$f(30) = 2^{30} = 1073741824$$

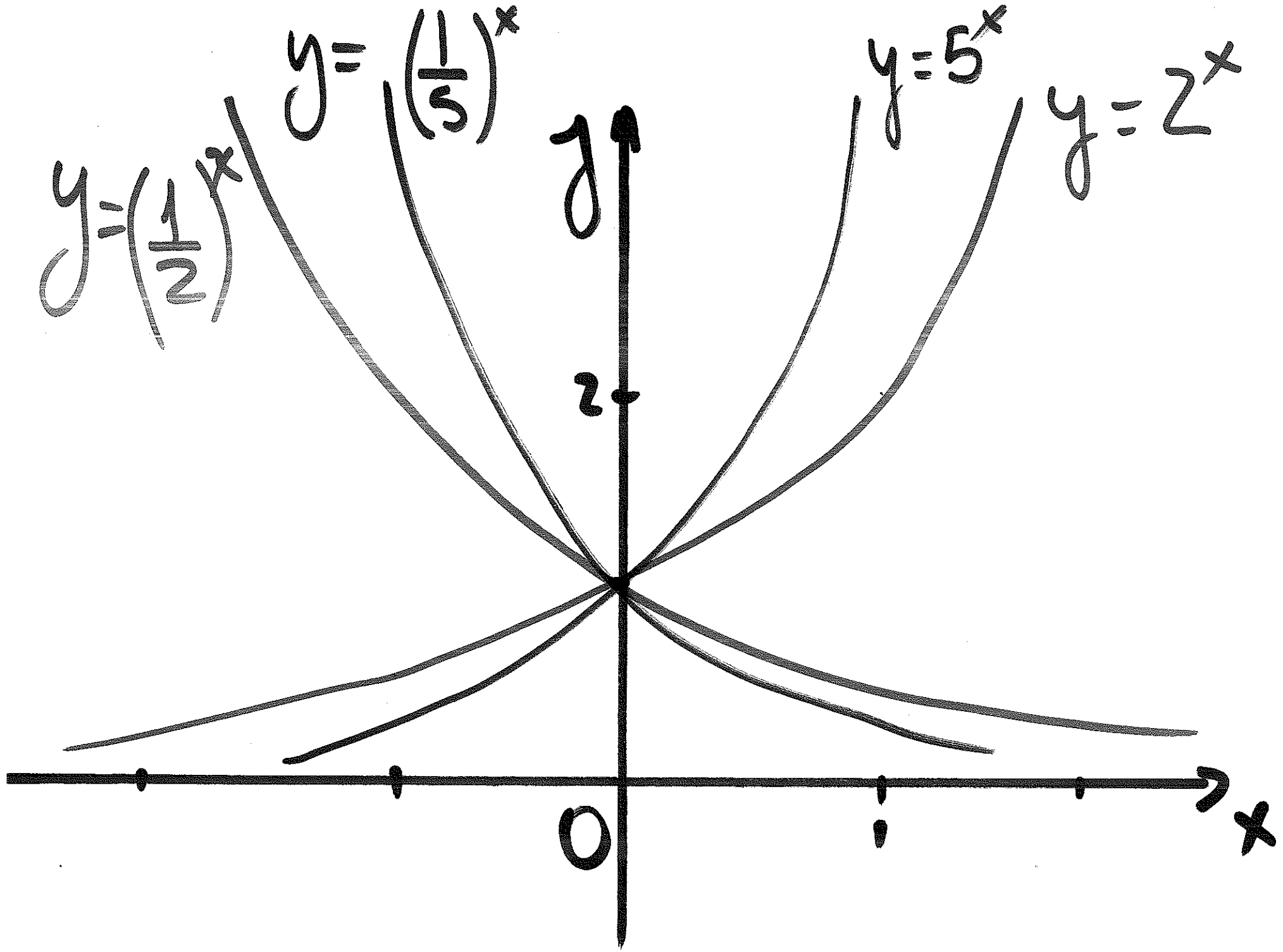

#bac
f(t)=3^t
1
0
a
b
t

$R \longrightarrow f(x) = \underbrace{a}_{\text{BASE}}^{x}$

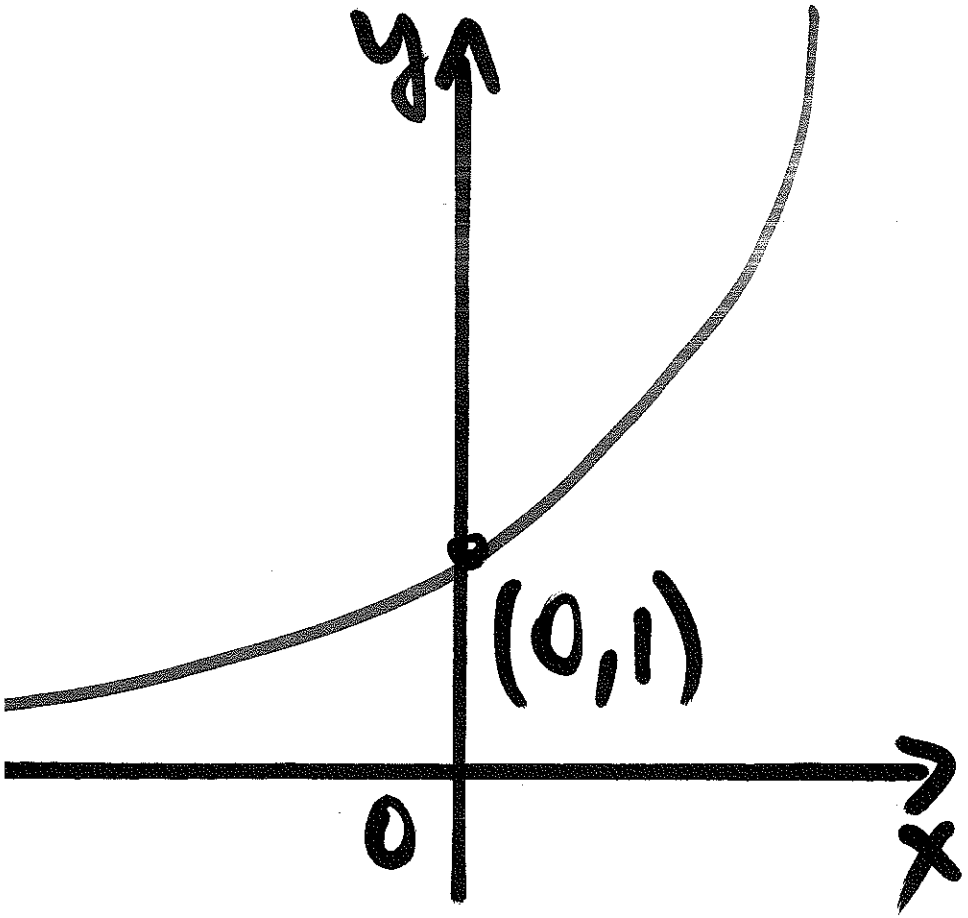
$a > 0$ y $a \neq 1$]

GRAFICAR

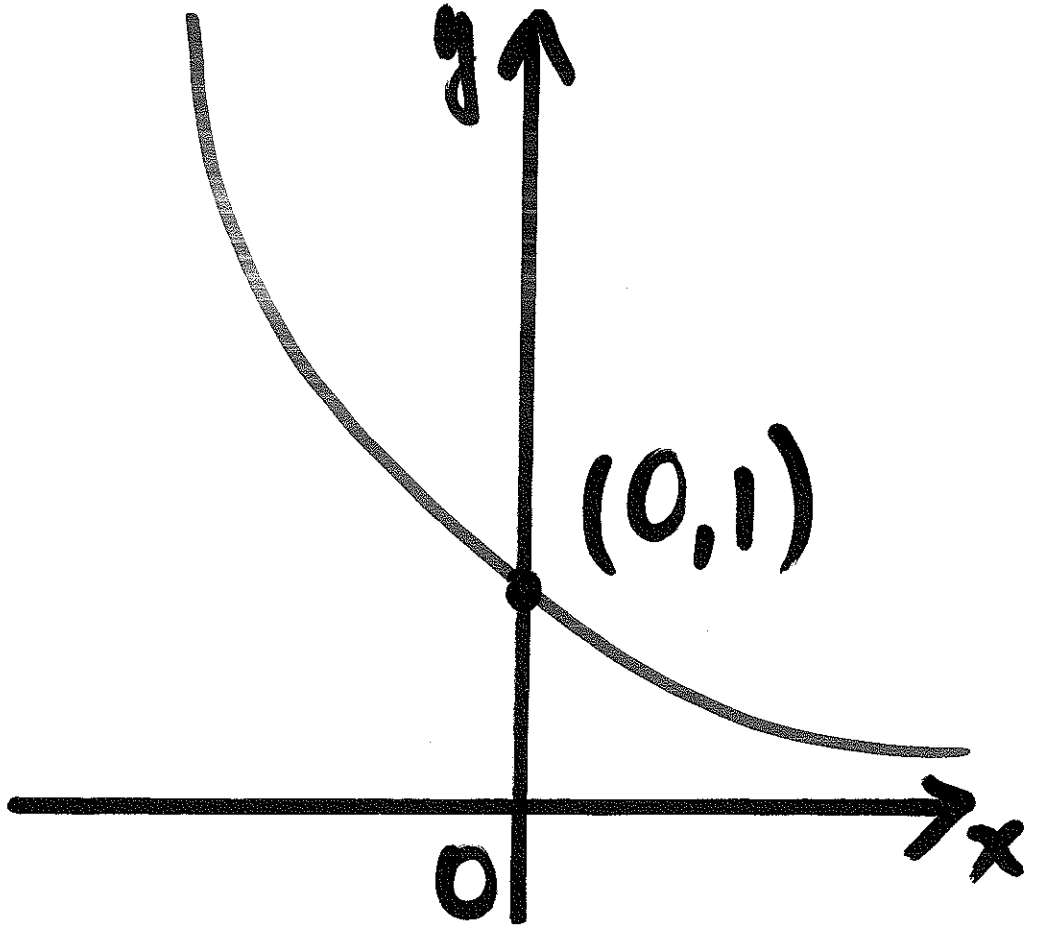
$\frac{1}{5}, \frac{1}{2}, 2, 5$


$y = \left(\frac{1}{2}\right)^x$
$y = \left(\frac{1}{5}\right)^x$
$y = 5^x$
$y = 2^x$
y
2
0
1
x

$f(x) = a^x \quad (a > 0, a \neq 1)$



$a > 1$



$0 < a < 1$

VALE

DESPEJE → $f(2) = 25$

$a^2 = 25$

$\boxed{a = 5}$

TRANSFORMACIONES

$g(x) = 1 + 2^x$ (DV)

$h(x) = -2^x$ (R)

$t(x) = 2^{x-1}$ (DH)

FUNCION EXPONENCIAL NATURAL

n	$\left(1+\frac{1}{n}\right)^n$
1	2
5	2.48832
100	2.70481
10 000	2.71815
1 000 000	2.71828

$\approx e$

FUNCION EXPONENCIAL NATURAL

$$f(x) = e^{x}$$

BASE

$f(x) = a^x \quad (a > 0 \text{ y } a \neq 1)$

→ biyectiva → tiene inversa

función logarítmica con base a → $\log_a$

Sea a # + con $a \neq 1$:

$$\log_a x = y \Leftrightarrow a^y = x$$

base

exponente

base

exponente

forma logarit.	forma expon.
$\log_{10} 100000 = 5$	$10^5 = 100000$
$\log_2 8 = 3$	$2^3 = 8$
$\log_2 \left(\frac{1}{8}\right) = -3$	$2^{-3} = \frac{1}{8}$
$\log_5 5 = 1$	$5^1 = 5$

PROPIEDADES

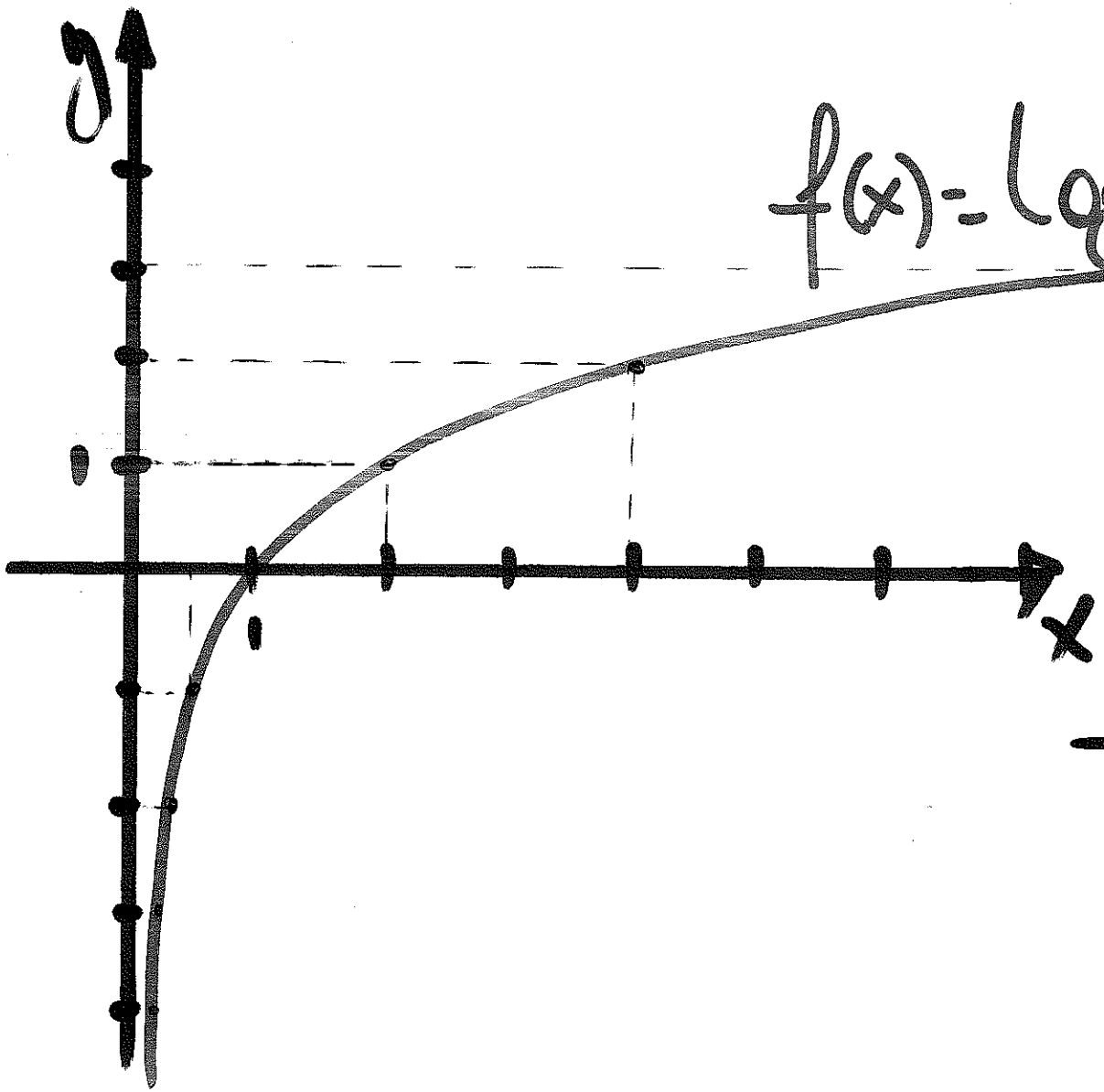
1. $\log_a 1 = 0 \rightarrow a^0 = 1$

2. $\log_a a = 1 \rightarrow a^1 = a$

3. $\log_a a^x = x \rightarrow a^x = a^x$

4. $a^{\log_a x} = x \rightarrow \log_a x = \log_a x$

Def



x	$\log_2 x$
2^2	2
2	1
1	0
2^{-1}	-1

$\log_a x$ → Reflexión
→ D. vertical
→ D. horizontal

Logaritmo Común

$$\log x = \log_{10} x$$

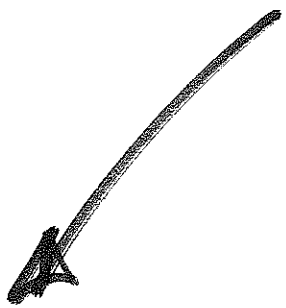
10 → base

$$\log 10 = 1$$

$$\log 100 = 2$$

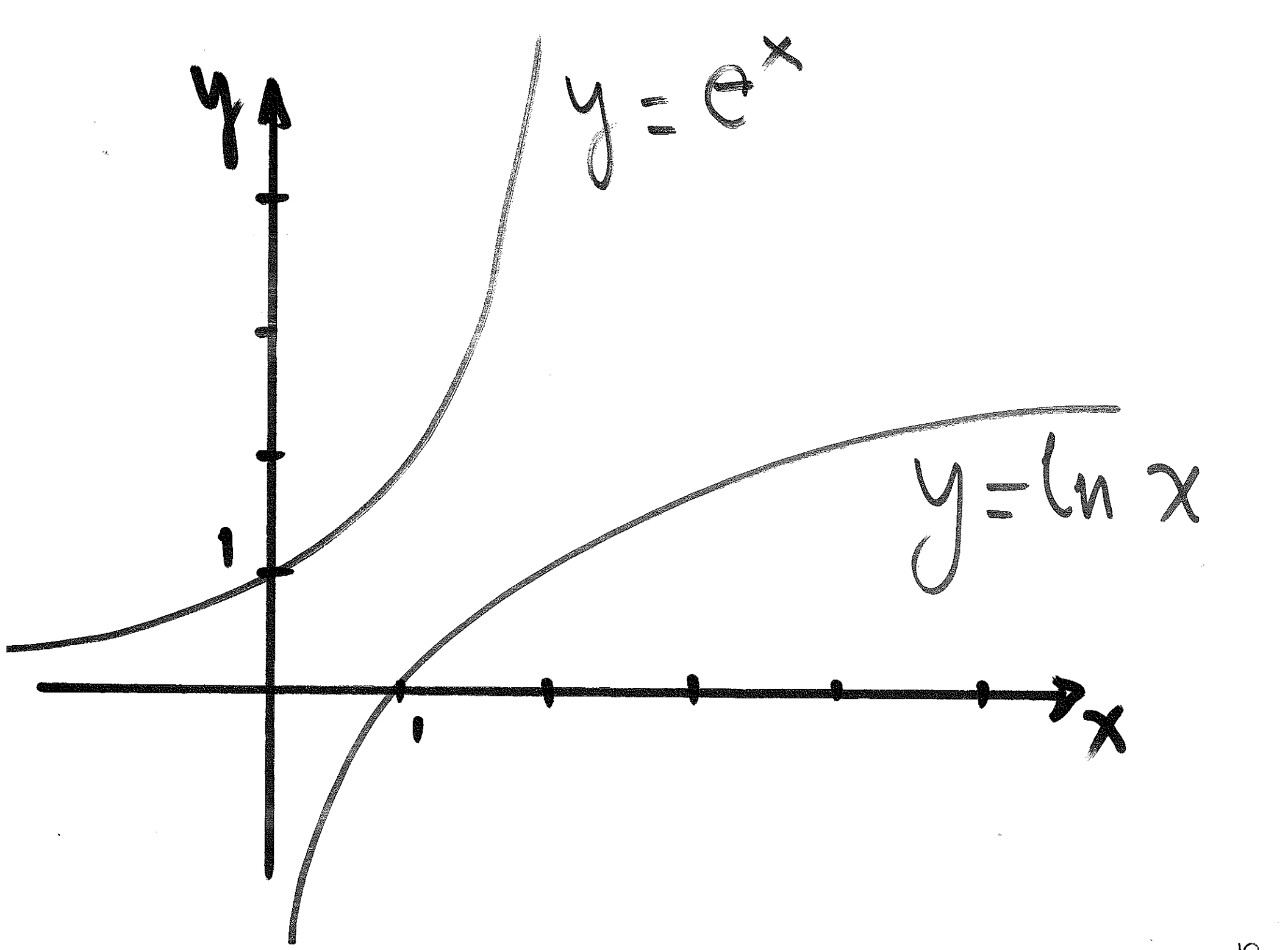
Logaritmo natural

$$\ln = \log_e x$$



base "e"

$$\ln x = y \Leftrightarrow e^y = x$$


y
y = e^x
y = ln x
1
1
x

PROPIEDADES ln

1. $\ln 1 = 0 \rightarrow e^0 = 1$
2. $\ln e = 1 \rightarrow e^1 = e$
3. $\ln e^x = x \rightarrow e^x = e^x$
4. $e^{\ln x} = x \rightarrow \ln x = \ln x$

· Def

Leyes

1. $\log_a(AB) = \log_a A + \log_a B$

2. $\log_a\left(\frac{A}{B}\right) = \log_a A - \log_a B$

3. $\log_a(A^C) = C \log_a A$

Ej:

$$\log_2(6x) = \log_2 6 + \log_2 x \quad \text{L1}$$

$$3\log x + \frac{1}{2}\log(x+1) =$$

$$= \log x^3 + \log(x+1)^{1/2} \quad \text{L3}$$

$$= \log\left(x^3(x+1)^{1/2}\right) \quad \text{L1}$$

¡ATENCION!

$$\log_a(x+y) \neq \log_a x + \log_a y$$

$$\frac{\log 6}{\log 2} \neq \log\left(\frac{6}{2}\right)$$

$$(\log_2 x)^3 \neq 3\log_2 x$$

Cambio de base

$$y = \log_b x$$ Exp

$$b^y = x$$ $\log_a$

$$\log_a(b^y) = \log_a x$$ L3

$$y \log_a b = \log_a x$$ Dist.

$$y = \frac{\log_a x}{\log_a b}$$

Cambio de base

$$\log_b x = \frac{\log_a x}{\log_a b}$$

Si $x = a \Rightarrow \log_b a = \frac{1}{\log_a b}$