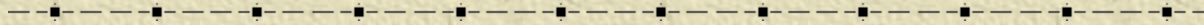
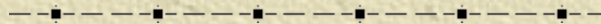


Fenómenos Críticos



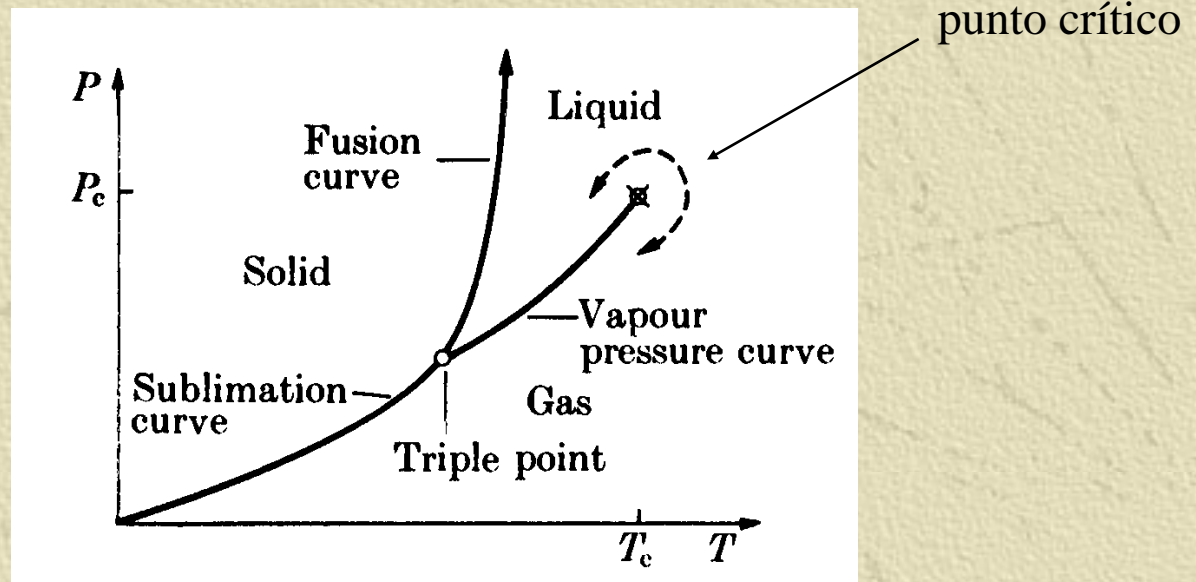
Autora de la presentación: Clarisa Lona



Transiciones de fase termodinámicas

- fenómenos de **equilibrio**
- **macroscópicos**

Ej: fluido normal

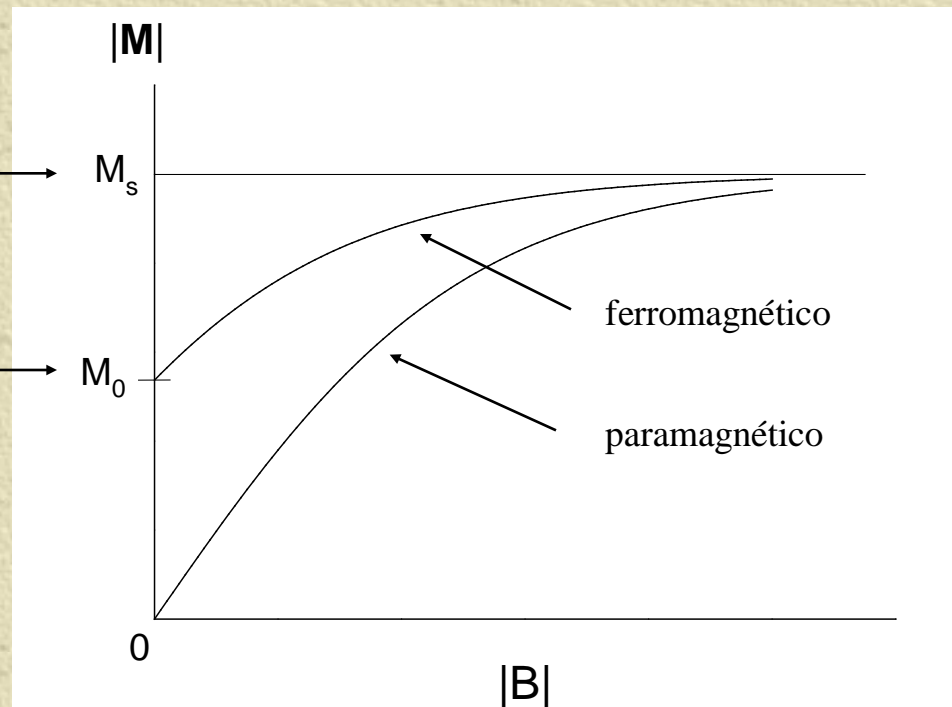


Ferromagnetismo en sólidos

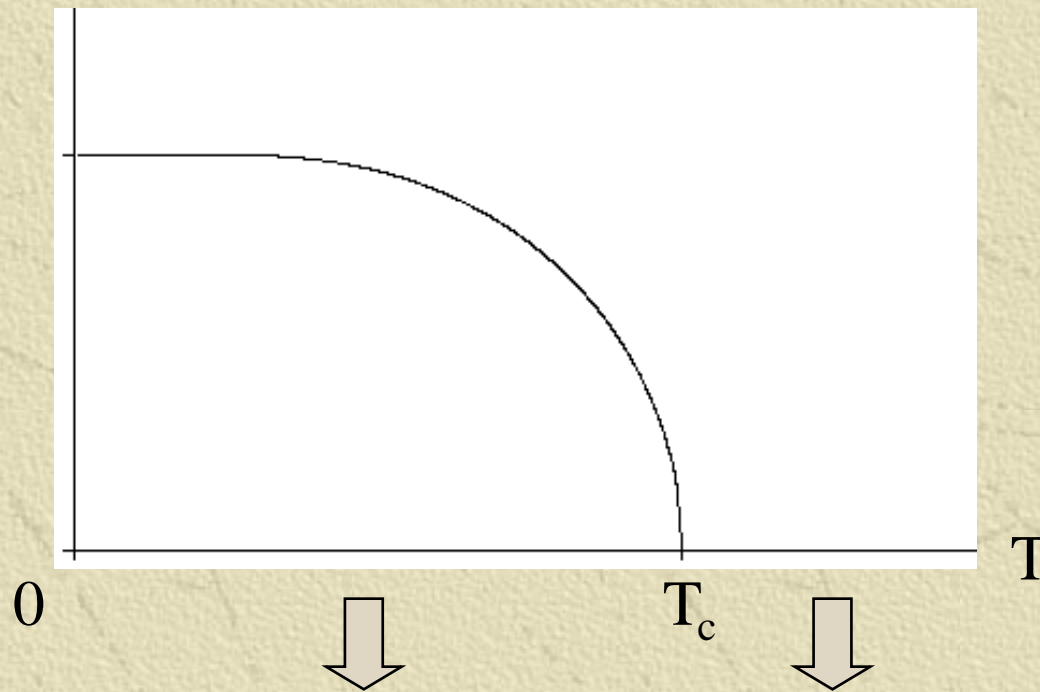
Magnetización \mathbf{M} \leftrightarrow campo magnético \mathbf{B}

Magnetización
de saturación

Magnetización
espontánea



$M_0(T)$



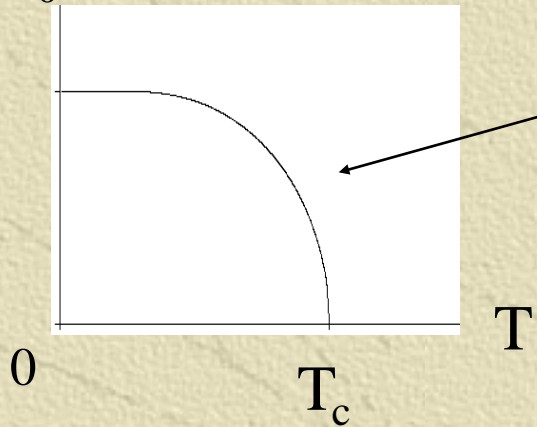
T_c : temperatura crítica

Fase ferromagnética

Fase paramagnética

$T = T_c$: transición de fase **continua**, de 2do orden ó
fenómeno crítico

$M_0(T)$



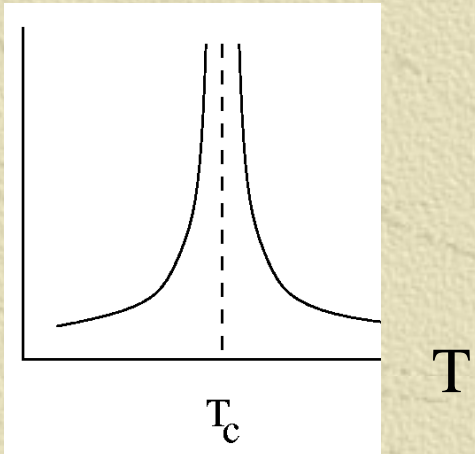
$$M_0(T) \sim (T_c - T)^\beta$$

$\beta > 0$: exponente crítico

Calor específico $C(T)$:

$$dQ = V C(T) dT; \quad C(T) = \frac{1}{V} \frac{dQ}{dT}$$

$C(T)$



$$C(T) \sim |T - T_c|^{-\alpha} \quad \text{para} \quad |T - T_c| \ll 1$$

$\alpha > 0$: exponente crítico

Respuesta ante un campo magnético externo B

Susceptibilidad magnética:

$$\chi(T) = \frac{1}{V} \left(\frac{\partial M(T, B)}{\partial B} \right)_{B=0}$$

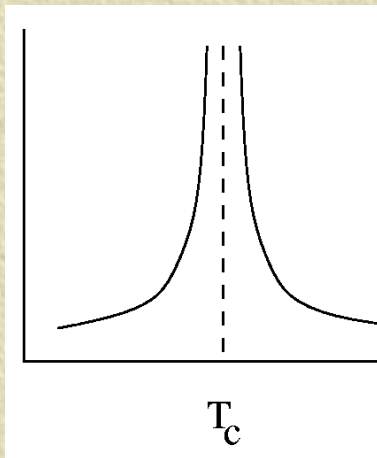
$$\delta M \approx \chi(T) B$$

para

$$V = 1$$

$$B \ll 1$$

$\chi(T)$



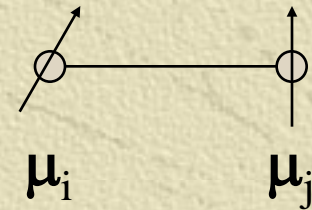
T

$$\chi(T) \sim |T - T_c|^{-\gamma} \quad \text{para} \quad |T - T_c| \ll 1$$

$\gamma > 0$: exponente crítico

Origen microscópico del ferromagnetismo

Interacción de intercambio: $E_{ij} = -J \boldsymbol{\mu}_i \cdot \boldsymbol{\mu}_j$



μ_i : momento magnético ó spin

Modelo de Ising



$$S_i = \pm 1$$



$$E = -J \sum_{\langle i,j \rangle} S_i S_j - B \sum_{i=1}^N S_i$$

Mecánica Estadística

Dada la energía de una configuración $\{S_1, S_2, \dots, S_N\}$ permite calcular promedios termodinámicos de cualquier cantidad que dependa de estas variables

Distribución de Boltzmann:

$$P(S_1, S_2, \dots, S_N) = \frac{e^{-E/T}}{\sum_{S_1, S_2, \dots, S_N} e^{-E/T}}$$

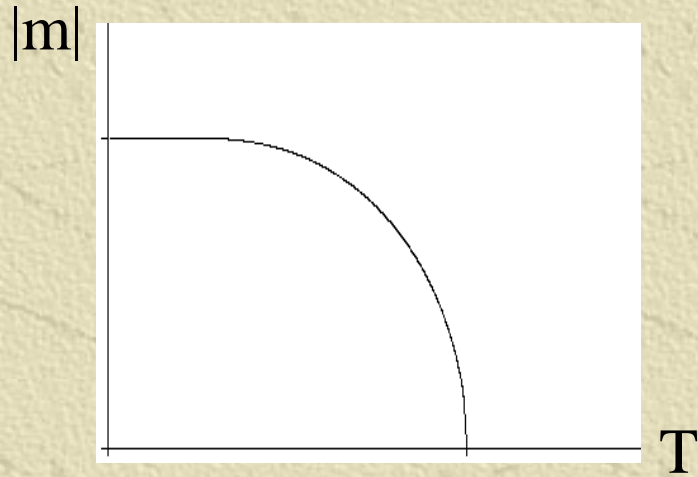
$$E = -J \sum_{\langle i, j \rangle} S_i S_j - B \sum_{i=1}^N S_i$$

Ej.: magnetización

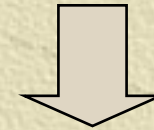
$$M(S_1, S_2, \dots, S_N) = \sum_{i=1}^N S_i$$

$$m(B, T) = \frac{1}{N} \sum_{S_1, S_2, \dots, S_N} P(S_1, S_2, \dots, S_N) M(S_1, S_2, \dots, S_N) = \frac{1}{N} \langle M \rangle$$

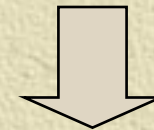
$B = 0$



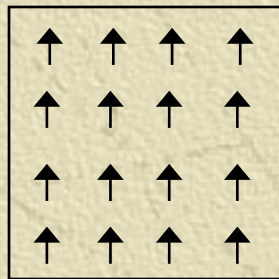
Transición solo ocurre
para $N \rightarrow \infty$



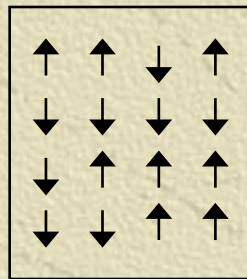
Fenómeno **cooperativo**,
auto-organizado



Sistema complejo!



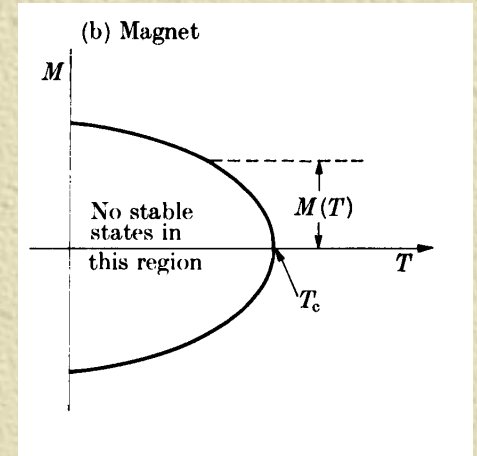
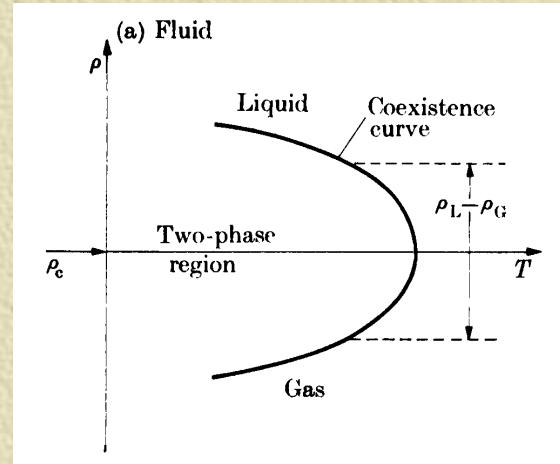
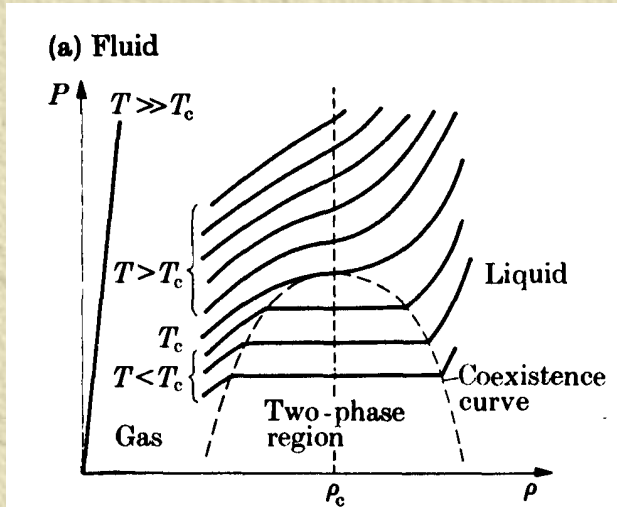
orden



desorden

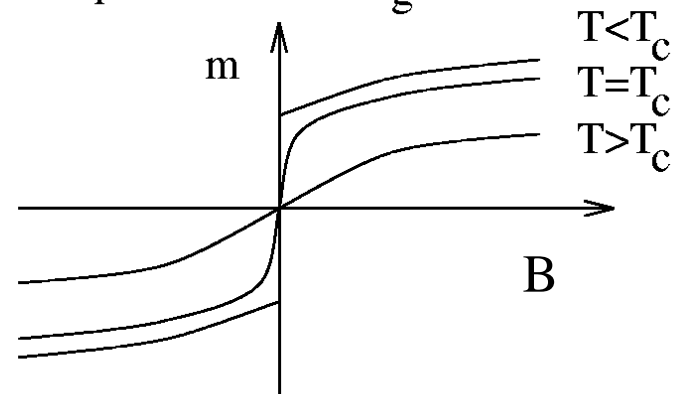
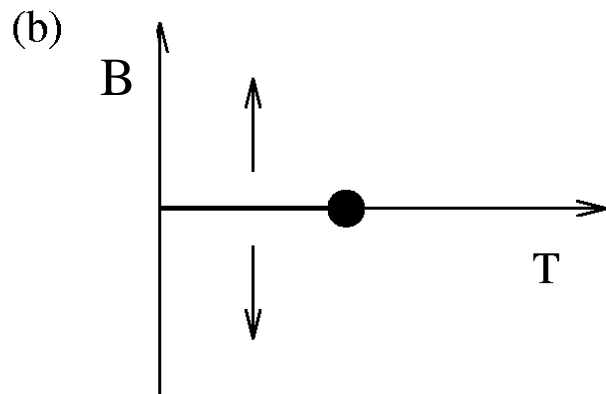
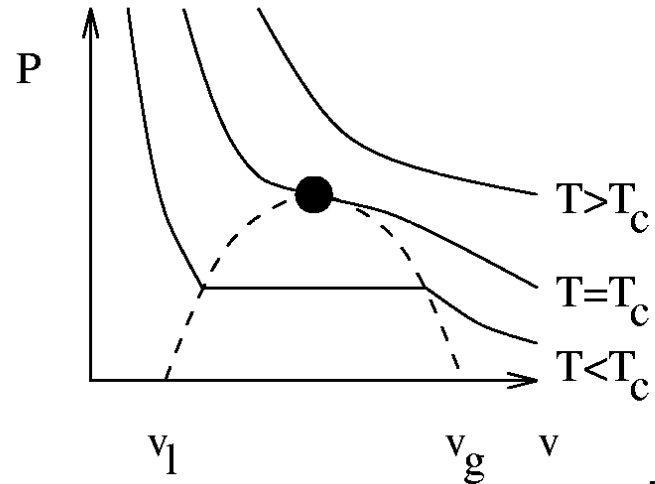
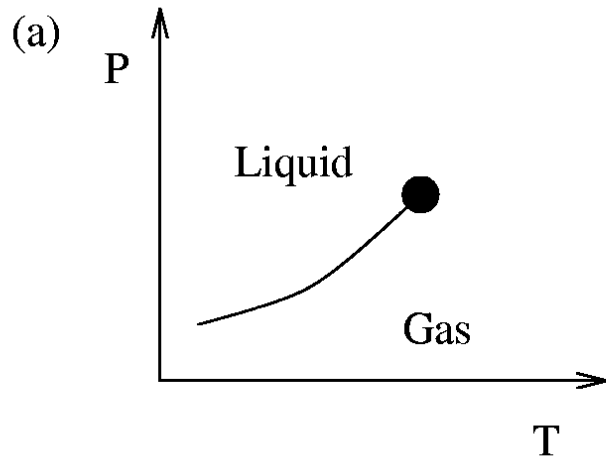
m : parámetro de orden

Fluidos vs. ferromagnetos



$$\rho_{L,G}(T) - \rho_C \sim \rho_{\pm} (T_c - T)^{\beta} \quad \text{para} \quad T_c - T \ll 1$$

$$C(T) \sim |T - T_c|^{-\alpha} \quad \text{para} \quad |T - T_c| \ll 1$$



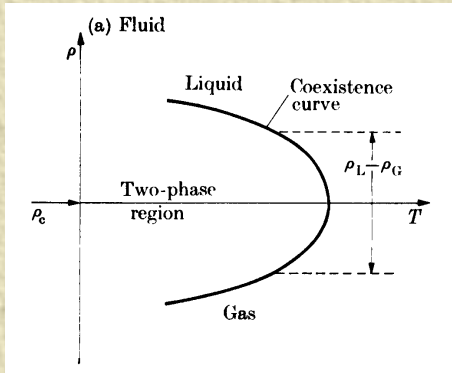
$$v = 1/\rho$$

susceptibilidad magnética: $\chi(T) = \frac{1}{V} \left(\frac{\partial M(T, B)}{\partial B} \right)_{B=0}$

$$\chi(T) \sim |T - T_c|^{-\gamma} \quad \text{para } |T - T_c| \ll 1$$

compresibilidad isotérmica: $\kappa_T(T, P) = \frac{1}{\rho} \frac{\partial \rho(T, P)}{\partial P}$

$$\kappa_T(T, P_c) \sim |T - T_c|^{-\gamma} \quad \text{para } |T - T_c| \ll 1$$



$$\rho_{L,G}(T) - \rho_c \sim \rho_{\pm} (T_c - T)^{\beta}$$

$$\frac{\rho_{G,L}(T)}{\rho_c} \sim 1 + \frac{\rho_{+,-} T_c^{\beta}}{\rho_c} \left(1 - \frac{T}{T_c}\right)^{\beta}$$

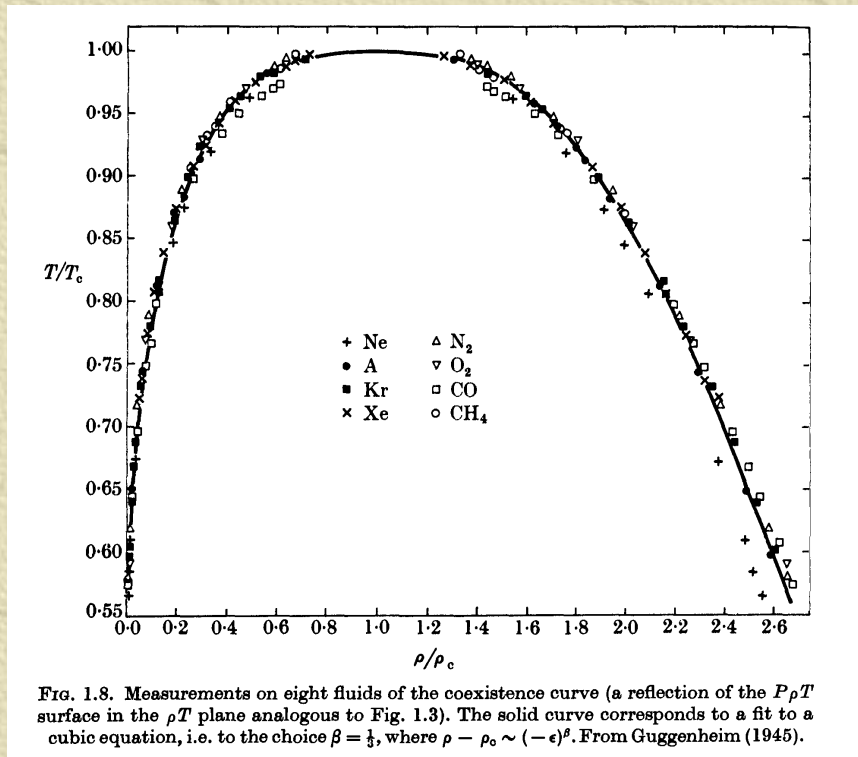
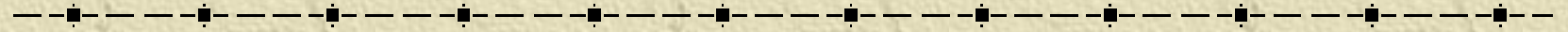


FIG. 1.8. Measurements on eight fluids of the coexistence curve (a reflection of the $P\rho T$ surface in the ρT plane analogous to Fig. 1.3). The solid curve corresponds to a fit to a cubic equation, i.e. to the choice $\beta = \frac{1}{3}$, where $\rho - \rho_c \sim (-\epsilon)^{\beta}$. From Guggenheim (1945).

Ley de estados correspondientes

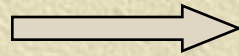
$$\beta \sim 1/3$$

Universalidad



exponente	Xe	Ni	fluido binario	bronce β	Ising d=3 (exacto)
α	< 0.2	0.04 ± 0.12	0.113 ± 0.005	0.05 ± 0.06	0.12
β	0.35 ± 0.015	0.358 ± 0.003	0.322 ± 0.002	0.305 ± 0.005	0.31
γ	1.3 ± 0.2	1.33 ± 0.02	1.239 ± 0.002	1.25 ± 0.02	1.25

Exponentes críticos



Clases de Universalidad

Que determina la clase de universalidad?

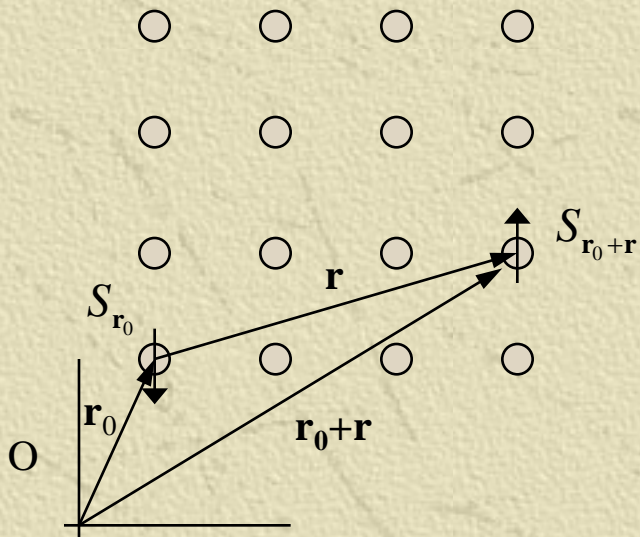
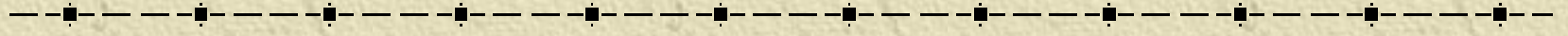
- d : dimensión espacial
- simetría del parametro de orden
- interacciones de corto alcance

Teoría: Grupo de renormalización

- ✦ Kenneth Wilson (1971) – Premio Nobel de Física 1982
- ✦ Leo P. Kadanoff (1965-1969)
- ✦ Michael E. Fisher (1964-1969)
- ✦ B. Widom (1965-1967)

Concepto principal: **INVARIANCIA DE ESCALA**

Correlaciones espaciales



$$C(r) = \frac{1}{N} \sum_{\mathbf{r}_0} \langle (S_{\mathbf{r}_0} - \langle S_{\mathbf{r}_0} \rangle) (S_{\mathbf{r}_0 + \mathbf{r}} - \langle S_{\mathbf{r}_0 + \mathbf{r}} \rangle) \rangle$$

correlación entre **fluctuaciones** de spines
a una distancia $r = |\mathbf{r}|$

para $T \neq T_c$ $C(r) \approx e^{-r/\xi}$

$\xi(T)$: longitud de correlación

para $T \rightarrow T_c$ $\xi(T) \approx |T - T_c|^{-\nu}$ $\nu > 0$: exponente crítico

para $T \sim T_c$ $C(r) \approx \frac{1}{r^{d-2+\eta}} e^{-r/\xi}$ $\eta > 0$: exponente crítico

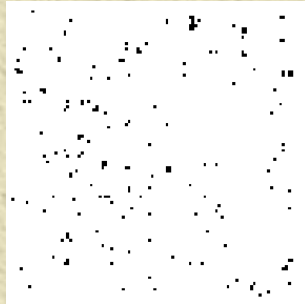
Sistemas críticos presentan **correlaciones espaciales de largo alcance!**



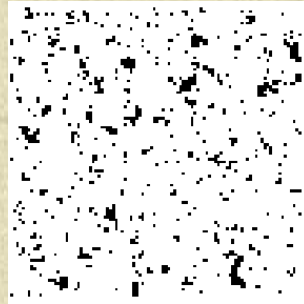
Extrema sensibilidad ante perturbaciones externas \rightarrow divergencia en las funciones respuesta

Ising d=2 – Simulaciones numéricas

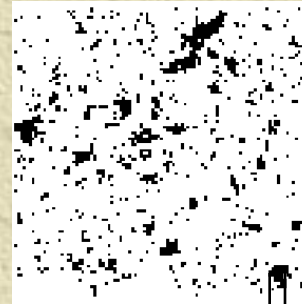
$T = 0.8 T_c$



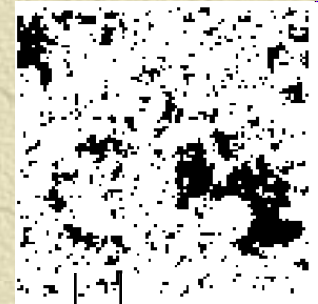
$T = 0.95 T_c$



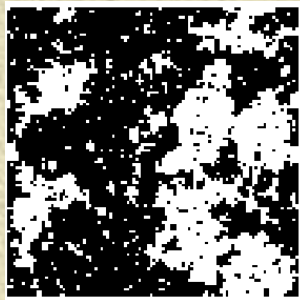
$T = 0.99 T_c$



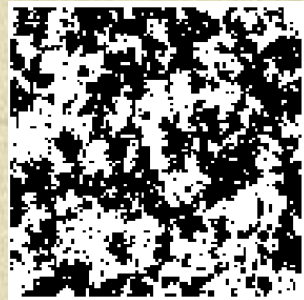
$T = 0.999 T_c$



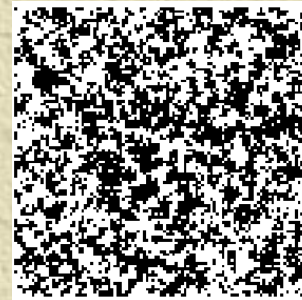
$T = T_c$



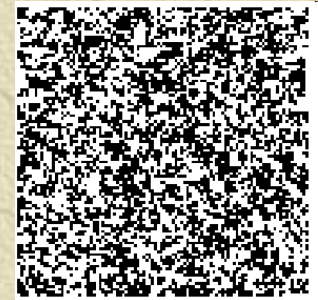
$T = 1.1 T_c$



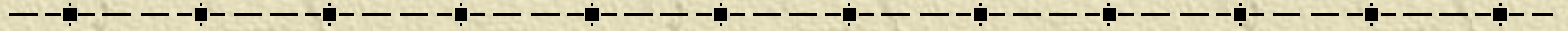
$T = 1.5 T_c$



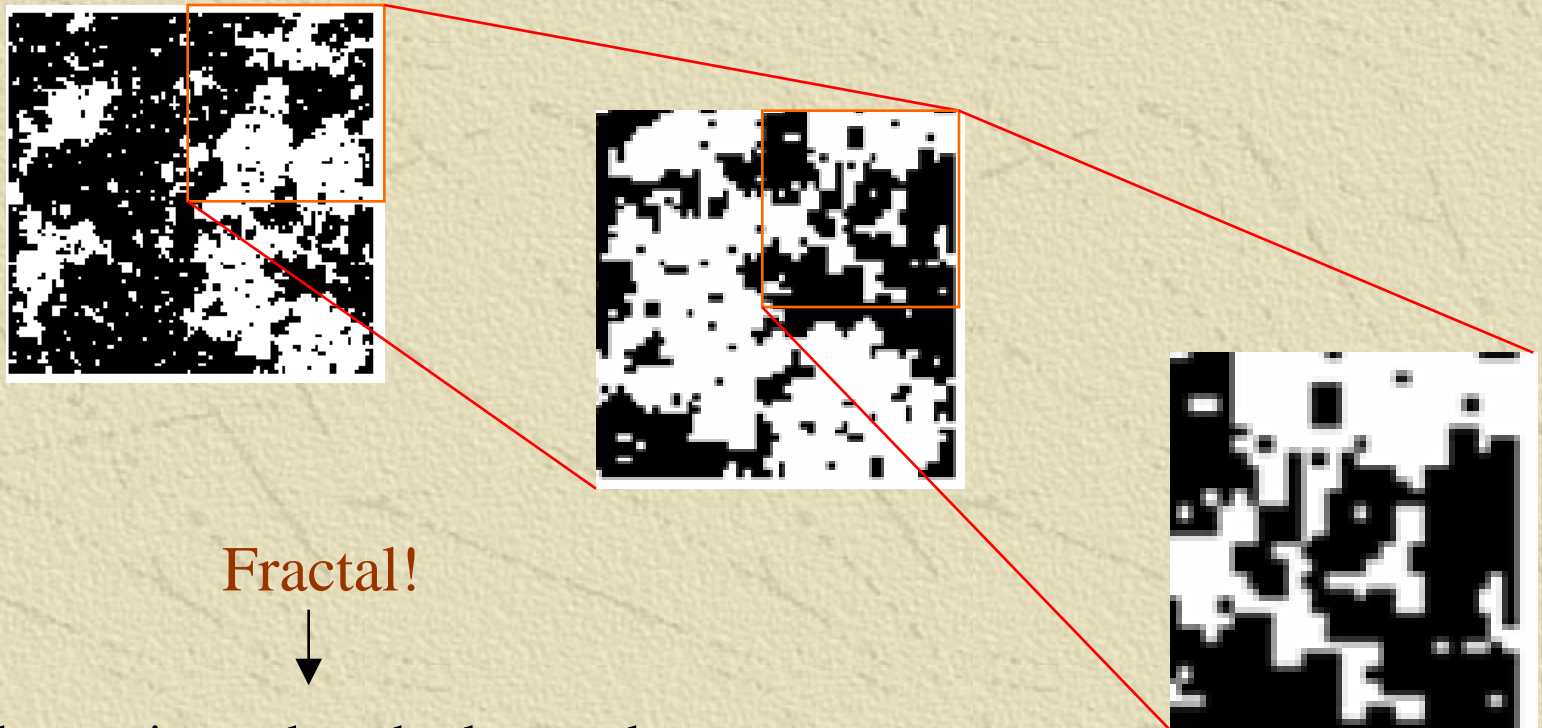
$T = 2 T_c$



INVARIANCIA DE ESCALA



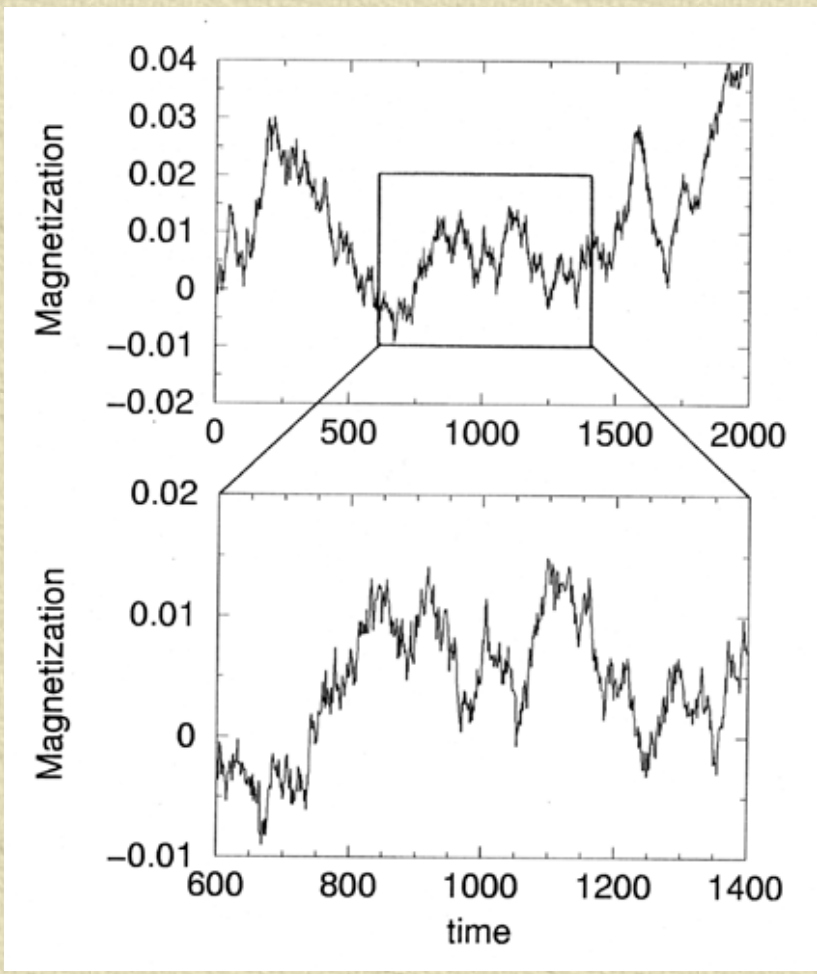
$$T = T_c$$



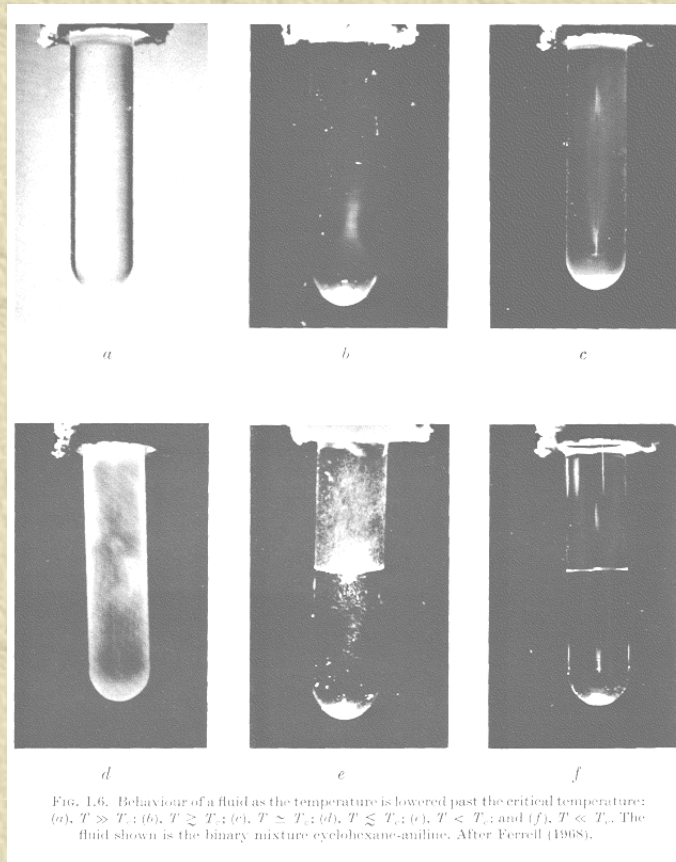
Fractal!



Fluctuaciones de todas las escalas



Opalescencia crítica



Grupo de renormalización

Conjunto de transformaciones de escala:



“ZOOM termodinámico” $T \rightarrow T_c$ $\xi \rightarrow \infty$



Universalidad: los exponentes críticos solo dependen de aquellas propiedades que permanecen invariantes ante una transformación de escala

Correlaciones temporales

Relajación al equilibrio:

$$m(t) = m_{eq}(T) + A e^{-t/\tau} \quad \text{para } T \neq T_c$$

$\tau(T)$: tiempo de relajación

frenado crítico: $\tau(T) \sim \xi^z \sim |T-T_c|^{-z\nu}$ $z > 0$: exponente crítico dinámico



Sistemas críticos presentan **correlaciones temporales de largo alcance!**

Fenómenos críticos

- ✦ **Invariancia de escala** no trivial (longitud de correlación divergente)
- ✦ Correlaciones **espaciales y temporales** (entre fluctuaciones) de **largo alcance** (leyes de potencia)
- ✦ Sensibles ante perturbaciones externas: funciones respuestas divergentes → leyes de potencia caracterizadas por exponentes críticos
- ✦ **Universalidad** → fenómenos independientes de los detalles finos